SCENARIO QNS ON LOGARITHMS, INDICES AND SURDS.

❖ *Indices-Focused Questions*

1. A local environmental agency is monitoring the rapid spread of an invasive algae species in Lake Victoria. The area covered by the algae, in square meters, is observed to triple every two weeks. Initially, a small patch of 3^4square meters was detected.

(a) Calculate the estimated area covered by the algae after 8weeks, expressing your answer using index notation and then as a numerical value.

(b) If the agency estimates the total surface area of the affected part of the lake to be 3^{10} square meters, determine after how many full weeks the algae will cover at least one-ninth of this estimated total area.

(c) The agency introduces a new bio-agent that reduces the algae coverage by15% each week. If they apply this agent when the algae covers 3^6 square meters, calculate the remaining area after 4 weeks, expressing your answer in standard form. Advise the agency on the effectiveness of this bio-agent over the long term.

3. A microbiologist is culturing a new strain of bacteria in a petri-dish.The number of bacteria doubles every30 minutes under optimal conditions. The initial count was 5×10^3 bacteria.

(a) Calculate the total number of bacteria after 4 hours, expressing your answer in standard form.

(b) Due to limited nutrients, the petri-dish can only sustain a maximum of 1.28×10^6 bacteria. Determine the maximum number of full 30-minute intervals before the bacterial population exceeds the dish's capacity.

(c) If a new antibiotic is introduced that reduces the bacterial population by a factor of $2^{-0.5}$ every hour, calculate the number of bacteria remaining from the initial count after 3 hours of antibiotic treatment.

❖ *Logarithms-Focused Questions*

4. Mr. Okello invests UGX5,000,000 in a savings account that offers an annual interest rate of 8% compounded continuously. He plans to use this investment to pay for his child's university fees.

a. Calculate the time, to the nearest month, it will take for Mr. Okello's investment to double, using the formula for continuous compounding ($A = Pe^{rt}$) and natural logarithms.

b. University fees are projected to be UGX15,000,000 in10 years. Determine if Mr. Okello's Initial investment will be sufficient. If not, calculate the additional amount he needs to invest today to reach the target, assuming the same continuous compounding rate.

c. Mr. Okello is also considering another investment option that offers 8.5% annual interest compounded quarterly. Advise him on which investment strategy would yield a higher return over a 5-year period, providing clear calculations and justification using logarithms.

5. A research team is studying a newly discovered radioactive substance, Isotope-X, for potential use in medical imaging. It decays exponentially. They measured its initial mass as100 grams, and after 5 days, the mass reduced to 70 grams.

(d) Determine the decay constant for Isotope-X, expressing your answer to three significant figures.

(e) Calculate the half-life of Isotope-X in days.

(f) The team needs to transport a sample of Isotope-X, but for safety, its mass must be less than 10grams. If they start with 100grams, calculate the minimum number of full days they must wait before transporting the sample.

6. A forest ranger is tracking the spread of a wild-fire. The area covered by the fire, $A(t)$ in hectares, can be modeled by the equation $A(t) = A_0e^{kt}$, where A_0 is the initial area and t is the time in hours. At the start of monitoring, $A_0 = 5$ hectares. After 2 hours, the fire covered 12 hectares.

a. Determine the growth constant k for the fire, expressing your answer to three significant figures.

b. The fire is considered out of control if it reaches100 hectares. Calculate the time, to the nearest hour, it will take for the fire to reach this size.

c. The ranger has access to a new fire retardant that can reduce the growth constant by 20%. If applied immediately, how much longer, to the nearest hour, would it take for the fire to reach100 hectares compared to without the retardant?

7. A chemist is preparing solutions for an experiment. The pH of a solution is defined as $pH = -\log_{10}[H^+]$, where $[H^+]$ is the hydrogen ion concentration in moles per liter.

a. A solution has a hydrogen ion concentration of 3.2×10^{-5}mol/L. Calculate its pH value to two decimal places.

b. The experiment requires a solution with a pH of 8.5.Calculate the hydrogen ion concentration required for this solution, expressing your answer in standard form.

❖ *Surds-Focused Questions*

8. A carpenter is building a custom triangular bookshelf for a client. The client provided specific dimensions for the base and height of the triangle, but some measurements involve irrational numbers due to precise aesthetic requirements.

(a) The base of the bookshelf is $(\sqrt{75} + \sqrt{12})$ cm and the height is $(\sqrt{48} - \sqrt{3})$ cm. Calculate the exact area of the triangular bookshelf in its simplest surd form.

(b) The carpenter needs to add a decorative trim along the hypotenuse. If the bookshelf is a right-angled triangle with the given base and height, calculate the exact length of the trim needed.

(c) To ensure stability, a diagonal brace is added. If the brace connects the midpoint of the base to the top vertex, calculate its exact length in simplest surd form.

9. A landscape architect is designing a garden with a complex pathway. The path involves several segments whose lengths are expressed as surds due to precise geometric requirements and natural stone placements.

(a) A section of the path is composed of three segments with lengths $2\sqrt{20}$ meters, $3\sqrt{45}$ meters, and $\sqrt{80}$ meters. Calculate the total exact length of this section of the path in its simplest surd form.

(b) Another section of the path is rectangular, with a length of $(4 + \sqrt{7})$ meters and a width of $(4 - \sqrt{7})$ meters. Calculate the exact area of this rectangular section.

(c) A circular fountain is to be placed in a square section of the garden. If the side length of the square is $(6+2\sqrt{3})$ meters, and the fountain's diameter is $(2+\sqrt{3})$ meters, calculate the exact area of the square region *excluding* the fountain, leaving your answer in simplest surd form.

10. An engineer is designing a high-precision component for a new machine. Several critical dimensions of this component are expressed as surds to maintain exact errors.

(a) A key length is given by $L1=\sqrt{128}-\sqrt{50}+\sqrt{18}$ mm. Simplify L1 to its simplest surd form.

(b) Another dimension is defined by the expression $\frac{5}{\sqrt{10}}$. Rationalize the denominator and simplify this expression.

(c) The area of a rectangular section of the component is $(3+\sqrt{2})$ cm^2, and its width is $(3-\sqrt{2})$ cm. Calculate the exact length of this section.

❖ *Integrated questions*

11. A pharmaceutical company is developing a new drug. The concentration of the drug in a patient's bloodstream, C(t), decreases over time t (in hours) according to the formula $C(t)=C_o e^{-kt}$, where C_0 is the initial concentration and k is the decay constant. The drug is administered via an injection that delivers an initial concentration of $2\sqrt{18}$ units/mL.

(a) Simplify the initial concentration C_0 to its simplest surd form.

(b) If the concentration drops to $C_0/2$ after 2 hours, calculate the decay constant k using natural logarithms, expressing your answer to three significant figures.

(c) The drug becomes ineffective when its concentration drops below 10^{-2} units/mL. Determine the maximum time (in hours) the drug remains effective, expressing your answer in standard form.

12. Mrs Brenda a biological scientist is monitoring the population of a rare insect species in a nature reserve. The population, P(t), after t years, is modeled by $P(t)=P_0(1+r)^t$, where P_0 is the initial population and r is the annual growth rate. The initial population was 10^5 insects. Due to habitat changes, the population is experiencing a slight decline.

(a) If the population decreased to 0.9×10^5 after one year, calculate the annual decay rate r.

(b) Mrs Brenda needs to determine the time it takes for the population to halve. As her biology student, calculate this time in years, expressing your answer using logarithms to two decimal places.

(c) A specific part of the habitat is a rectangular area with dimensions $(\sqrt{200}-\sqrt{8})$ km by $(\sqrt{50}+\sqrt{2})$ km. Calculate the exact area of this habitat in its simplest surd form. Explain how this area influences the population model and the scientist's efforts.

13. A civil engineer is designing a new bridge. The stress on a particular support beam, S, is modeled by $S=K\cdot e^{-\alpha x}$, where K is a constant, α is a material property, and x is the distance from the load point. The maximum allowable stress is 10^3 units.

(a) If $K=5\times10^4$ and $\alpha=\ln(2)$, calculate the distance x (to two decimal places) at which the stress reduces to 5×10^3 units.

(b) The length of a critical component in the beam is given by $L=\frac{3\sqrt{12}}{\sqrt{3}+\sqrt{2}}$. Rationalize the denominator and simplify L to its exact surd form.

(c) If the material property α is found to be $2^{\left(\frac{1}{2}\right)}$, express this value in its simplest surd form and discuss how this might affect the stress reduction compared to the previous α value, requiring an understanding of indices and surds.

*****GOOD LUCK*****

"If you want to go fast, go alone. If you want to go far, go together"